Cambridge Pre-U

**Cambridge Assessment International Education**
Cambridge Pre-U Certificate

**FURTHER MATHEMATICS** **9795/02**

Paper 2 Further Application of Mathematics **May/June 2019**

MARK SCHEME

Maximum Mark: 120

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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This document consists of **16** printed pages.



Cambridge Assessment International Education

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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|---|
| GENERIC MARKING PRINCIPLE 5:<br><br>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen). |
| GENERIC MARKING PRINCIPLE 6:<br><br>Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | 35 | **B1** | |
| | 35 | **B1** | Not √35 or $35^2$ |
| 1(b) | N(35, 35) | **B1ft** | Mean and variance as in **(a)** |
| | $r_{min} \geqslant$ "35" + $z$√"35" – 0.5 **oe** | **M1** | Ignore ±, but not just – |
| | $z$ = 2.05(4) | **B1** | Independent. Condone 2.06 |
| | ($r_{min} \geqslant$) 46.6(5) | **A1** | Can be implied |
| | ⇒ $r_{min}$ = 47 **cso** | **A1** | From 46.6(5) and proper working which correct deals with continuity correction |
| | **Alternative 1** | | |
| | W′~N(35, 35) | **B1ft** | Mean and variance as in **(a)** |
| | Consideration of $\Phi^{-1}(0.98)$ for N(35, 35) | **M1** | |
| | 47.15... | **A1** | BC |
| | So P(W′ < 47.5) > 0.98 | **A1** | Can be implied e.g. by 0.9827 |
| | ⇒ $r_{min}$ = 47 **cso** | **A1** | Must be complete and correct argument including cc consideration |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(b) | **Alternative 2** | | |
| | $W' \sim N(35, 35)$ | **B1ft** | Mean and variance as in **(a)** |
| | Consideration two values of $P(W' < w)$ for two different values of $w$. | **M1** | One prob $\geqslant$ 0.98 and the other $\leqslant$ 0.98 |
| | $P(W' < 46.5) = 0.9740...$ | **A1** | Other values need detailed |
| | $P(W' < 47.5) = 0.9826...$ | **A1** | cc consideration for final A1 |
| | $\Rightarrow$ $r_{min} = 47$ **cso** | **A1** | Must be complete and correct argument including cc consideration |
| 2(a) | Weeds must occur independently<br>and at constant average rate | **B1** | At least one must be contextualised |
| | | **B1** | Do not allow ‘uniform rate’ or ‘constant rate’<br>Allow explanation, e.g. ‘Whether a weed occurs in one position mustn’t affect whether another appears anywhere else’. Ignore statements of conditions (e.g. ‘singly’) provided not incorrect or contradictory.<br>Not ‘the number of weeds is independent’ or ‘the probability of weeds is independent’. |
| 2(b) | Po(4) | **M1** | Stated or implied |
| | 1 – 0.8893 | **M1** | Allow M1M1 for 0.0511 |
| | = 0.1107 | **A1** | Answer, awrt 0.111 |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(c) | $\lambda = 3.60$, $P(\leqslant 6) = 0.9267$ | **B1** | Inequality may not be explicit but probability must be linked to correct $\lambda$ value |
| | $\lambda = 3.70$, $P(\leqslant 6) = 0.9182$ | **B1** | |
| | $\Rightarrow \quad 3.60 < \lambda < 3.70$ | **M1** | Forming finite inequality in $\lambda$ or $5A$ from $\lambda$ values |
| | $\Rightarrow \quad 0.72 < A < 0.74$ | **A1** | This answer only. If units shown then must be correct |
| 3(a) | Unbiased *or* allows conclusions *or* means sample more likely to represent population | **B1** | Or similar |
| 3(b) | $13.4 \pm z\sqrt{\dfrac{4.63}{40}} \left( \text{or } 13.4 \pm t\sqrt{\dfrac{4.63}{40}} \right)$ | **M1** | 40 and $z$-(or $t$-)value both needed |
| | $z$ = awrt 1.96 (or $t$ = awrt 2.02) | **A1** | |
| | (12.73, 14.07) (or (12.71, 14.09) from $t$-distribution) | **A1** | 2 dp specifically demanded |
| 3(c) | Yes as not told distribution of times was normal | **B1** | Yes with reason which must not imply that the parent distribution is (approximately) normal. |
| 3(d) | $\overline{y} = 18.2$ | **B1** | |
| | $\hat{\sigma}_y^2 = 7.627...$ | **M1** | 7.5: M1A0 |
| | | **A1** | Awrt 7.63, can be implied |
| | $18.2 - 13.4 \pm (z\,/\,t)\sqrt{\dfrac{4.63}{40} + \dfrac{7.627}{60}}$ with $z = 1.96$ or $t$ in [1.98, 2.00] | **M1** | Correct structure |
| | | **A1ft** | FT $z$ as in **(b)** |
| | (3.83, 5.77) (or (3.82, 5.78) from $t$-distribution) | **A1** | End points correct to 3 sf |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(e) | Yes, samples need to be independent | **B1** | |
| 4(a) | $\int_0^2 \frac{1}{4}\pi\sin(\frac{1}{2}\pi t)\,\mathrm{d}t = \left[-\frac{1}{2}\cos(\frac{1}{2}\pi t)\right]_0^2$ | **M1** | Attempt to integrate, limits |
| | | **B1** | Correct indefinite integral |
| | $= -½(-1-1)$<br>$= 1$ and justify this as satisfying a condition | **A1** | Correctly show area = 1 |
| | $\mathrm{f}(t) \geqslant 0$ for all $t$ in interval $0 \leqslant t \leqslant 2$ | **B1** | Explicitly stated |
| 4(b) | 1 | **B1** | No working needed |
| 4(c) | $\int_0^{1-w} \frac{1}{4}\pi\sin(\frac{1}{2}\pi t)\,\mathrm{d}t = 0.05$<br>$\left[-\frac{1}{2}\cos(\frac{1}{2}\pi t)\right]_0^{1-w} = 0.05$ | **M1** | Integral, any relevant limits and equate to consistent value |
| | $\cos(\frac{1}{2}\pi(1-w)) = 0.9$ *or* $\sin(\frac{1}{2}\pi w) = 0.1$ | **A1** | Solve as far as correct equation involving single unknown sin or cos term. |
| | $w =$ **awrt** 0.713 | **A1** | |
| 4(d) | $\int_0^2 \frac{\pi}{4}\cos^2(\frac{1}{2}\pi t)\sin(\frac{1}{2}\pi t)\,\mathrm{d}t$<br>$= \left[-\frac{1}{6}\cos^3(\frac{1}{2}\pi t)\right]_0^2$ | ***M1** | This integral or equivalent |
| | | **dep*M1** | Method for integral |
| | | **A1** | Correct indefinite integral |
| | $= \frac{1}{3}$ | **A1** | |
| 5(a) | Positive even integers | **B1** | Or ‘2, 4, 6, …’ |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | P(exactly one 6) = $\frac{10}{36}$ | **B1** | Any convincing explanation |
| | $\frac{13}{18}\times\frac{5}{18}=\frac{65}{324}$ or 0.2006… | **B1** | Exact or awrt 0.201 |
| 5(c) | $\frac{5}{18}t^2+\frac{13}{18}\times\frac{5}{18}t^4+(\frac{13}{18})^2\times\frac{5}{18}t^6+\ldots$ | ***M1** | Correct expression (must clearly be infinite sum |
| | $=\frac{5}{18}t^2\left(\frac{1}{1-\frac{13}{18}t^2}\right)=\frac{5t^2}{18-13t^2}$ **AG** | **dep*M1** | Use $S_\infty$ of GP |
| | | **A1** | Correctly obtain AG, at least one intermediate step needed |
| 5(d) | $\left(\frac{5t^2}{18-13t^2}\right)^2=\frac{25t^4}{324}\left(1-\frac{13}{18}t^2\right)^{-2}$ | **M1** | Square PGF |
| | | **M1** | Rearrange to obtain $(1+x)^{-2}$ |
| | $=\frac{25t^4}{324}\left(1+\frac{13}{9}t^2+\frac{(-2)(-3)}{2}(\frac{13}{18}t^2)^2\right)$ | **M1** | Identifying $t^8$ term as required |
| | $\Rightarrow$coefficient of $t^8=\frac{4225}{34992}=0.1207\ldots$ | **A1** | Exact or a.r.t. 0.121, www |
| 6(a) | A random variable $T$ for which | **B1** | State random variable |
| | $\mathrm{E}(T)=\theta$ | **B1** | State expected value = $\theta$. |
| 6(b) | $\frac{\sigma^2}{n}$ | **B1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(c) | $[(X-\bar{X})+(\bar{X}-\mu)]^2 =$ $(X-\bar{X})^2+2(X-\bar{X})(\bar{X}-\mu)+(\bar{X}-\mu)^2$ | **M1** | Use $A^2 + 2AB + B^2$ |
| | $\mathrm{E}\left[(X-\bar{X})(\bar{X}-\mu)\right]=0$ | **B1** | Used |
| | $\mathrm{E}[(X-\mu)]^2 = \mathrm{E}(X-\bar{X})^2+\mathrm{E}(\bar{X}-\mu)^2$ | **A1** | Obtain this |
| | $\sigma^2 = \mathrm{E}(X-\bar{X})^2+\dfrac{\sigma^2}{n}$ | **M1** | Substitute results from **(b)** |
| | | **A1 ft** | Correct formula, ft on **(b)** |
| | $\mathrm{E}(X-\bar{X})^2=\sigma^2-\dfrac{\sigma^2}{n}\Rightarrow$ **AG** | **A1** | Correctly obtain given answer. (May award without B1 but must have previous A marks) |
| 6(d) | $\dfrac{n}{n-1}(X-\bar{X})^2$ or $\dfrac{(X-\bar{X})^2}{1-\frac{1}{n}}$ | **B1** | |
| 7(a) | $W_e - W_f = \tfrac{1}{2}\times 700(25^2-15^2)$ | **M1** | Signs wrong: M1A0 |
| | $= 140\,000$ | **A1** | |
| 7(b) | $W_e = 140\,000 + 150\times 100 + 250\times 100$ | **M1** | 3 terms needed. ft *their* ‘140 000’ |
| | $= 180\,000$ | **A1** | Signs wrong: M1 A0 |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | $\uparrow$: $T\cos\theta + T\cos\varphi - 0.04g = 0$ | **M1** | One equation, 3 terms |
| | $\rightarrow$: $T\sin\theta + T\sin\varphi = 0.04 \times 0.6\omega^2$ | **M1** | Second equation, same $T$ |
| | | **A1** | Both equations correct. Mark only awarded once values substituted |
| | $\cos\theta = \sin\varphi$ and vice versa. | **B1** | Used. Or $\cos\dfrac{\theta+\phi}{2} = \sin\dfrac{\theta+\phi}{2}\left(=\dfrac{1}{\sqrt{2}}\right)$ |
| | $\dfrac{\sin\theta+\sin\phi}{\cos\theta+\cos\phi} = \dfrac{0.04\times0.6\omega^2}{0.04g}$ | ***M1** | Eliminating $T$ to leave equation in $\omega$, $v$ or time-period, possibly involving $\theta/\varphi$ |
| | $0.04 \times 0.6\omega^2 = 0.04g$ | **dep*M1** | Dealing with angle and solving |
| | $\omega = \sqrt{\dfrac{g}{0.6}} \approx 4.08 \text{ rad s}^{-1}$ | **A1** | |
| 8(b) | $T\cos\theta + T\cos\varphi = 0.4$ | **M1** | Substitute into equation from **(a)** |
| | $0.6T + 0.8T = 0.4$ | **A1** | Correct sin/cos $\varphi$ |
| | $T = \dfrac{2}{7}$ or **awrt** 0.286 | **A1** | In range (0.285, 0.286), allow $\frac{2}{7}$ |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a) | $v_y = u_y$ (= $u \sin \theta$) | M1 | Velocity \|\| unchanged (may be in words or inferred from diagram) |
| | $v_x = -0.6u_x$ (= $-0.6u \cos \theta$) | M1 | Velocity ⊥ reversed and × $e$ . Reversal <u>must</u> be evident algebraically |
| | $w_x = v_x = -0.6u \cos \theta$ or $-0.6u_x$ | A1 | One final component correct |
| | $w_y = -ev_y = -0.6u \sin \theta$ or $-0.6u_y$ | A1 | Other final component correct<br>Note: For above 4 marks: Condone use of 'vertical'.<br>Must explicitly consider two collisions in two directions<br>Angles and other symbols <u>must</u> be unambiguously defined or evident from reasonable diagram. |
| | $\mathbf{w} = \begin{pmatrix} -0.6u\cos\theta \\ -0.6u\sin\theta \end{pmatrix}$ or $\begin{pmatrix} -0.6u_x \\ -0.6u_y \end{pmatrix} = -0.6\mathbf{u}$<br>so $k = 0.6$ | A1 | Or <u>complete</u> and fully correct argument using angles<br>Correctly deduce AG <u>using vectors</u> or careful argument with components with $k = 0.6$ stated (not just from magnitudes)<br>Mark can only be awarded following M1M1A1A1<br>Note: Finding $w = 0.6u$ alone is insufficient for final A1 |
| 9(b) | $\mathbf{I} = m(\mathbf{w} - \mathbf{u})$ | M1 | Use impulse = $\Delta(m\mathbf{v})$ (or $\pm 0.224\cos\theta$ from 1st coll$^n$) |
| | $\|\mathbf{I}\| = 0.2(0.7 - (-0.6 \times 0.7))$ | A1 | Correct signs (or $\pm 0.224\sin\theta$ from 2nd coll$^n$) |
| | = 0.224 | A1 | Answer 0.224 or $\frac{28}{125}$ |
| | In direction of **w** | B1 | Direction correctly stated, any recognisable form |
| 10(a) | M($A$): $\sqrt{3} \times 12 - 2R_B = 0$ | M1 | Take moments about any point (must be clear attempt at $Fd$)<br>If $d = 1$ not shown point must be clearly specified |
| | ⇒ $R_B = 6\sqrt{3}$ or 10.4 N | A1 | Correct value of one normal component of contact force |
| | $R_A = R_B$ (= 10.4 N) | A1ft | Component of force at other point same, ft on their answer. If no indication of directions given then assume correct but if directions indicated (e.g. on diagram) then must be correct/consistent or clearly explained in a final answer) |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(b) | M($A$): $2T\cos 30° = 2 \times 12 \sin 90°$ | **M1*** | Moments ($Fd$) about any point†<br>(condone $=\sqrt{3} \times 12 \cos 30°$ for M1)<br>If $d = 1$ not shown point must be clearly specified |
| | $\Rightarrow \quad T = 8\sqrt{3}$ or 13.9 N | **A1** | Correct value of tension |
| | N2(→): $N = T$ **soi** | **B1** | H/V forces |
| | N2(↓): $F = 12$ **soi** | **B1** | V/H forces<br><br>(For B1B1 either 2 perp forces at A or one force split into components) |
| | $F \leqslant \mu N \Rightarrow \quad \mu \geqslant \dfrac{12}{8\sqrt{3}}$ | **dep*M1** | Attempt to solve equations and correct use $F \leqslant \mu N$ or $F = \mu N$ |
| | $\Rightarrow \quad \mu_{\min} = \frac{1}{2}\sqrt{3}$ or 0.866 | **A1** | Correct value or awrt 0.866 |
| | | **A1** | Correct value asserted as minimum<br>May mention $\mu = \mu_{\min}$ (or in words) earlier in solution |
| 11(a) | Radial: $R + mg\cos\theta = mv^2/r$ | ***M1** | Apply NII radially (may be implied by $mg = mv^2/r$) with weight and $a_r$<br>If just $mg\cos\theta = mv^2/r$ withhold A0A0 unless carefully explained |
| | Setting $R = 0$, $r = 0.4$ and $\theta = 0$ to find $v^2$ (= 4) [$v = 2$] | **dep*M1** | Attempt to apply at top. Condone small errors (e.g. $r = 0.8$). $v^2$ may already be eliminated by energy. |
| | $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + 2mgr$ | **M1** | Proper use of conservation of energy (not general) to eliminate $v$ (must be used in NII equation) |
| | $\frac{1}{2}mu^2 = 2m + 2 \times 4m$ | **A1** | Use conservation of energy to derive a correct equation, no $v$ |
| | [$u^2 = 20$,] $u = \sqrt{20}$ or **awrt** 4.47 | **A1** | Not just $u \geqslant \sqrt{20}$ |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(b) | $a_r = (-)g\cos\theta = (-)6$ | **M1** | Resolve $mg$ either $\parallel$ or $\perp$ |
| | $a_t = g\sin\theta = 8$ | **A1** | One correct component |
| | | **A1** | Other correct component<br>Ignore all signs<br>Must be specified (i.e. 6, 8 A0A0) |
| 11(c) | $a_r = r\dot{\theta}^2$ , $a_t = r\ddot{\theta}$ | **M1** | Both formulae stated or implied |
| | so $\dot{\theta} = (-)3.87 \text{ rad s}^{-1}$ | **A1** | Allow $\sqrt{15}$. Allow $\omega$ for $\dot{\theta}$ |
| | $\ddot{\theta} = 20 \text{ rad s}^{-2}$ | **A1** | Ignore all signs<br>If not specified, M1A1A0 |
| 12(a) | $\dfrac{dv}{dt} = \dfrac{k(u^2 - v^2)}{2u}$ | **B1** | Equate expression to $dv/dt$ |
| | $\displaystyle\int \frac{2u}{u^2 - v^2}\,dv = \int k\,dt$ | **M1** | Separate correctly |
| | $\displaystyle\int \frac{1}{u+v} + \frac{1}{u-v}\,dv = \int k\,dt$ | **M1** | Factorise and use PFs |
| | $kt + c = \ln\left\|\dfrac{u+v}{u-v}\right\|$ or<br>$\ln\|u+v\| - \ln\|u-v\|$ | **A1** | Correct integral with constant<br>(condone changed constant if not critical) |
| | $\dfrac{u+v}{u-v} = Ae^{kt}$ | ***M1** | Exponentiation with constant correctly dealt with |
| | $u + v = (u - v)Ae^{kt}$<br>$\Rightarrow v(1 + Ae^{kt}) = uAe^{kt} - u$ | **dep*M1** | Attempt to rearrange to $v = f(t)$ (no major algebraic errors) |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 12(a) | $v = u\left(\dfrac{Ae^{kt}-1}{Ae^{kt}+1}\right)$ or $u\left(\dfrac{A'e^{kt}+1}{A'e^{kt}-1}\right)$ or $u\left(\dfrac{e^{kt}+A''}{e^{kt}-A''}\right)$ or $u\left(1-\dfrac{2}{Ae^{kt}+1}\right)$ oe | A1 | cao without wrong working<br>If integration done with limits then no A marks so max 5/7 (GS requested)<br>SC4/7 for $kt+c = 2\ln\left\lvert\sec\left(\sin^{-1}\dfrac{v}{u}\right)+\tan\left(\sin^{-1}\dfrac{v}{u}\right)\right\rvert$<br>*or* $2\ln\left\lvert\tan\left(\dfrac{1}{2}\sin^{-1}\dfrac{v}{u}+\dfrac{1}{4}\pi\right)\right\rvert$<br>correctly derived |
| | **Alternative** | | |
| | $\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{k(u^2-v^2)}{2u}$ | B1 | Equate expression to d$v$/d$t$ |
| | $\displaystyle\int \frac{2u}{u^2-v^2}\,\mathrm{d}v = \int k\,\mathrm{d}t$ | M1 | Separate correctly |
| | $2\tanh^{-1}\left(\dfrac{v}{u}\right) = kt + c$ | A1 | From formula book. Must have + $c$. |
| | $v = u\tanh\left(\dfrac{kt+c}{2}\right)$ | A1 | Rearranging to $v$ =<br>Note: only 4 marks available for this solution since it is not complete (valid only for $\lvert v\rvert < u$) |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 12(b)(i) | $A - 1 = 0$ | **M1** | Can be implied or found in **(a)**<br>If unexponentiated constant $c = 0$ found then mark is awarded on correct exponentiation.<br>ft for M1 but only on function of the form $v = u\left(\frac{ae^{pt}+b}{ce^{pt}+d}\right)$ oe with an arbitrary constant (non-zero constants) |
| | $v = u\left(\frac{e^{0.2t}-1}{e^{0.2t}+1}\right)$ | **A1** | $k$ must be replaced for A1 |
| | **Alternative** | | |
| | $c = \tanh^{-1} 0 = 0$ | **M1** | Can be implied but only from GS which <u>must</u> be of the form $v = u\tanh(at+c)$ |
| | $v = u\tanh(0.1t)$ | **A1** | |
| 12(b)(ii) | $e^{0.2t}$ dominates so top ≈ bottom<br>(or $\tanh 0.1t \to 1$ as $t \to \infty$) | **B1** | Not just $e^{0.2t} \to \infty$ |
| | $v \to u$ | **B1** | Condone e.g. ‘$v$ becomes $u$’ or $v = u$. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 12(c)(i) | $A + 1 = 2(A - 1)$ | **M1** | Or: $A = -3$ from $A - 1 = 2(A + 1)$<br>Expect to see $v = u\left(\dfrac{-3e^{-0.2t} - 1}{-3e^{-0.2t} + 1}\right)$ or $-u\left(\dfrac{1 + 3e^{-0.2t}}{1 - 3e^{-0.2t}}\right)$ oe<br>ft for M1 but only on function of the form $v = u\left(\dfrac{ae^{pt} + b}{ce^{pt} + d}\right)$ oe with an arbitrary constant (non-zero constants) |
| | $v = u\left(\dfrac{3e^{-0.2t} + 1}{3e^{-0.2t} - 1}\right)$ | **A1** | $k$ must be replaced for A1<br>Note: $\left\lvert\dfrac{u+v}{u-v}\right\rvert = 3e^{-0.2t} \Rightarrow \dfrac{u+v}{u-v} = 3e^{-0.2t}$ is an error here<br>(since $u - v < 0$) and leads to $v = u\left(\dfrac{3e^{-0.2t} - 1}{3e^{-0.2t} + 1}\right)$ for M1A0 |
| 12(c)(ii) | $v \to \infty$ as $3e^{-0.2t} \to 1$ | **M1** | i.e. their denominator set to 0 |
| | Model breaks down for $t \geqslant T$<br>where $3e^{-0.2T} = 1$ | **A1** | Other correct explanations allowed |
| | $T = 5 \ln 3 = 5.493\ldots$ | **A1** | Exact or awrt 5.49. Must be correct final statement (could be in words). |